



Introduction

Global fossil fuel reserves are depleting and climate change impacts are increasing, encouraging world energy supply to transition to renewable energy.

This project aims to model UK renewable energy future growth trends utilising grey models, analysing if the UK is on course to meet formal emissions targets of net-zero by 2050.



Grey Prediction

Grey systems are those which represent incomplete information, developed to solve uncertain system problems by processing uncertainty, multiple inputs, discrete data and incomplete data and converting it into a model for forecasting: this is the 'grey prediction' [1], [3], [4]. The traditional grey prediction model GM(1,1) can be constructed as:

Define all of the data as the original series:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\} \quad (1)$$

Perform one Accumulated Generation Operation (AGO) to add together the established original series and obtain the following series:

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}, \text{ where } x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), k = 1, 2, 3 \dots, n \quad (2)$$

GM(1,1) expresses a first-order differential equation with one variable:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \quad \text{where } b \text{ is a constant.} \quad (3)$$

The derivative definition leads to:

$$\frac{dX^{(1)}(t)}{dt} \cong X^{(1)}(k+1) - X^{(1)}(k) = X^{(0)}(k) \quad (4)$$

Additionally, $X^{(1)} \cong Z^{(1)}$

$$Z^{(1)}(k) \cong \frac{X^{(1)}(k) + X^{(1)}(k+1)}{2}, k = 2, 3, 4 \dots, n \quad (5)$$

Thus, the traditional standard form of the GM(1,1) model is:

$$X^{(0)}(k) + aZ^{(1)} = b \quad (6)$$

Nonlinear Grey Bernoulli Model (NGBM1,1)

The NGBM(1,1) is a prediction model derived by combining the GM(1,1) with the basic Bernoulli differential equation which allows higher prediction precision to fit nonlinear data from small sample sizes [1], [3], [4]. The NGBM(1,1) has been successfully applied in research such as when forecasting energy, economic and industry growth.

NGBM(1,1) Application

China consumes more energy and produces more greenhouse gas emissions than any other country [3]. By analysing the accuracy of the previously researched NGBM(1,1) for China's renewable energy consumption trends, viability of using NGBM(1,1) to model UK growth trends can be confirmed.

The first steps follow GM(1,1) (1) and (2). Use the Bernoulli equation to establish the NGBM differential equation:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b [X^{(1)}]^r \quad (7)$$

Substitute GM(1,1) (3), (4) and (5) into (7) to obtain the following NGBM difference equation:

$$X^{(0)}(k) + aX^{(1)}(k) = b [Z^{(1)}(k)]^r, k = 2, 3, 4 \dots, n \quad (8)$$

Where a and b are obtained using the least squares method with the NGBM differential and difference equations. Use the grey differential equation to obtain the grey accumulation equation:

$$\hat{X}^{(1)}(k+1) = \left[[X^{(0)}(1)]^{(1-r)} - \frac{a}{1-r} e^{-a(1-r)k} + \frac{b}{a} \right]^{\frac{1}{1-r}}, k = 1, 2, 3 \dots, n \quad (9)$$

Reduce (9) by using the inverse AGO to obtain the demand model:

$$\hat{X}^{(0)}(k) = \hat{X}^{(1)}(k) - \hat{X}^{(1)}(k-1), k = 1, 2, 3 \dots, n \quad (10)$$

These calculations were computed on MATLAB.

Forecast Accuracy

The forecast accuracy was initially tested by applying the model to the first $n/2$ data points to examine if it produced the remaining data points [3].

Forecast accuracy was further examined by calculating: the mean absolute error (MAE), the mean square error (MSE) and the mean absolute percentage error (MAPE). These are expressed as follows:

$$MAE = \frac{\sum |e|}{n}, MSE = \frac{\sum e^2}{n-1}, MAPE = \frac{\sum \left| \frac{e}{a} \right|}{n}$$

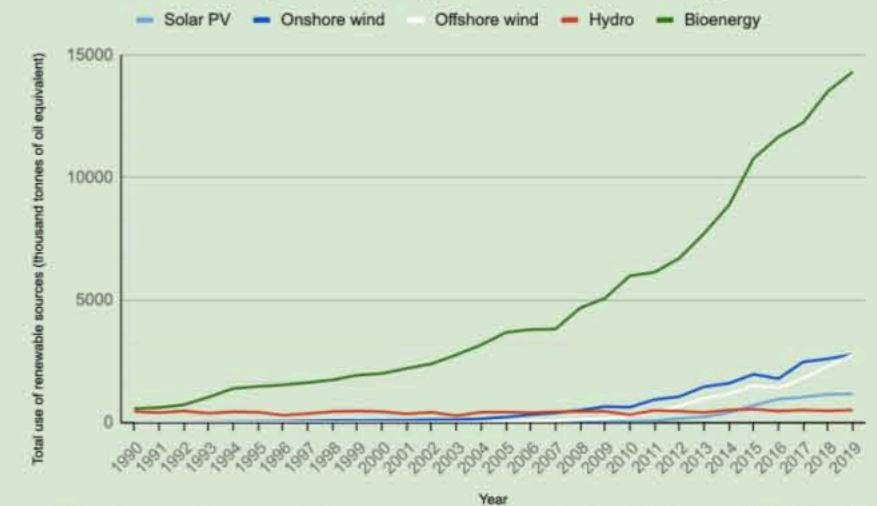
where e is the difference between the forecasted and actual value (a), n is the total number of predictions. In this case, the GM(1,1) MAE was 0.394%, MSE was 0.244% and MAPE was 5.855%. Meanwhile for NGBM(1,1), MAE was 0.333%, MSE was 0.186% and MAPE was 4.893%, indicating higher forecast accuracy.



Comparison between GM(1,1) and NGBM(1,1) model results for China's renewable energy consumption from 1991-2015.

UK Growth Trends

The NGBM(1,1) can be applied to the UK's raw data to extrapolate renewable energy growth trends tending to 2030, with less accurate but still indicative trends tending to 2050 [2]. This prediction data will be key for governments, businesses and civil society strategizing to meet net-zero targets.



Total use of renewable energy sources in the UK from 1990-2019.

Future Work

Once the NGBM(1,1) for the UK renewable energy data set is computed on MATLAB, the growth model for each energy source will be displayed with the appropriate graph.

NGBM(1,1) can be further optimised into novel hybrids such as the rolling NGBM(1,1), the Nash NGBM(1,1) or the weighted NGBM(1,1) [1], [4].

Parameters such as UK region and consumer demand demographics (homes, industry) can be added to gain a wider perspective on renewable energy growth trends. Additionally, comparing predicted UK growth with that of other countries allows visualisation of the global leaders in the race to net-zero.

References

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