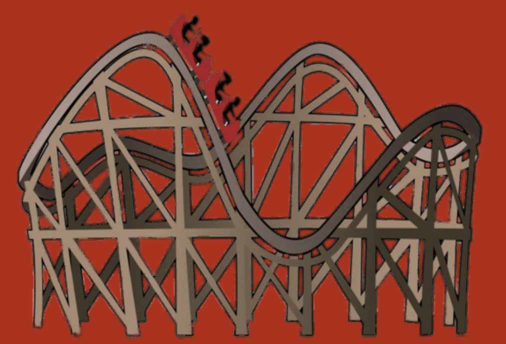

Data-driven Computational Mechanics and Machine Learning

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Introduction

In this project we discuss some results on data-driven computational mechanics and learn some basic techniques of machine learning. Consider computing the stress and strain of bars in a truss structure. We will investigate both the model-driven approach (used in conventional computing paradigms) and the data-driven approach. Since the data-driven approach uses experimental data directly instead of calibrating an empirical model, this method is more versatile and flexible. Hence, how can this approach be made efficient using machine learning (ML)?

Data-driven vs. model-driven

Data-driven computational mechanics is a new computing paradigm where calculations are formulated directly from experimental material data. Hence, we can disregard the empirical material modelling step of conventional computing.

The *model-driven* approach has been to calibrate empirical material models using observational data and then using the calibrated model for all other situations. However, this adds error and uncertainty to solutions (especially in systems with high-dimensional phase space or complex behaviour) due to:

- scatter and noise in the experimental data.
- inadequate knowledge of material laws and the phase space in which they are defined.

A simple truss structure

Suppose a simple elastic truss structure consisting of 3 hinged bars is stretched by forces (f_1, f_2) at point B and produces infinitesimal displacements (u_1, u_2).

Let (l_i, A_i, V_i) denote the lengths, cross-sectional areas and volumes of the 3 bars. We will determine the strain ε (extension per unit length) and stress σ (force per unit cross-sectional area) in the bars.

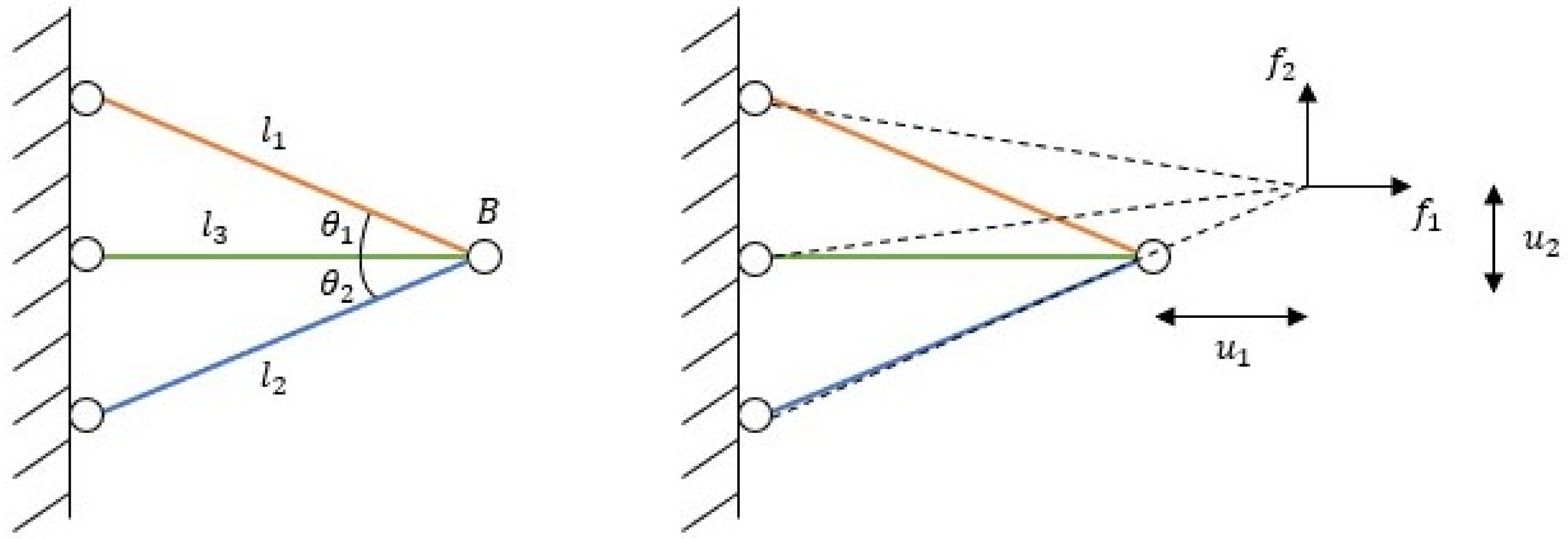

Figure 1: A simple truss structure before and after the application of forces at point B.

For simplicity, we assume that:

- forces only act at the hinges/joints.
- each bar only supports extension/compression in the truss structure.
- the simple system has two degrees of freedom, (i.e. horizontal and vertical displacements) u_1 and u_2.

Kinematics:

$$\varepsilon_1 \approx \frac{u_1}{l_1}\cos\theta_1 + \frac{u_2}{l_1}\sin\theta_1, \quad \varepsilon_2 \approx \frac{u_1}{l_2}\cos\theta_2 + \frac{u_2}{l_2}\sin\theta_2, \quad \varepsilon_3 \approx \frac{u_1}{l_1\cos\theta_1}$$

Force balance:

$$f_1 = \frac{V_1}{l_1}\sigma_1\cos\theta_1 + \frac{V_2}{l_2}\sigma_2\cos\theta_2 + \frac{V_3}{l_1\cos\theta_1}\sigma_3, \quad f_2 = -\frac{V_1}{l_1}\sigma_1\sin\theta_1 + \frac{V_2}{l_2}\sigma_2\sin\theta_2$$

Simplifying the above, we obtain:

$$\varepsilon_e = \sum_{i=1}^{2} B_{ei}u_i, \qquad f_i = \sum_{e=1}^{3} V_e B_{ei}\sigma_e, \qquad (1)$$

with $B_{11} = \frac{\cos\theta_1}{l_1}$, $B_{12} = \frac{\sin\theta_1}{l_1}$, $B_{21} = \frac{\cos\theta_2}{l_2}$, $B_{22} = \frac{\sin\theta_2}{l_2}$, $B_{31} = \frac{1}{l_1\cos\theta_1}$, $B_{32} = 0$.

Constitutive equation - Hooke's Law:

We use the simple model $\sigma_e = E_e\varepsilon_e$, where E_e is the Young's modulus for each bar.

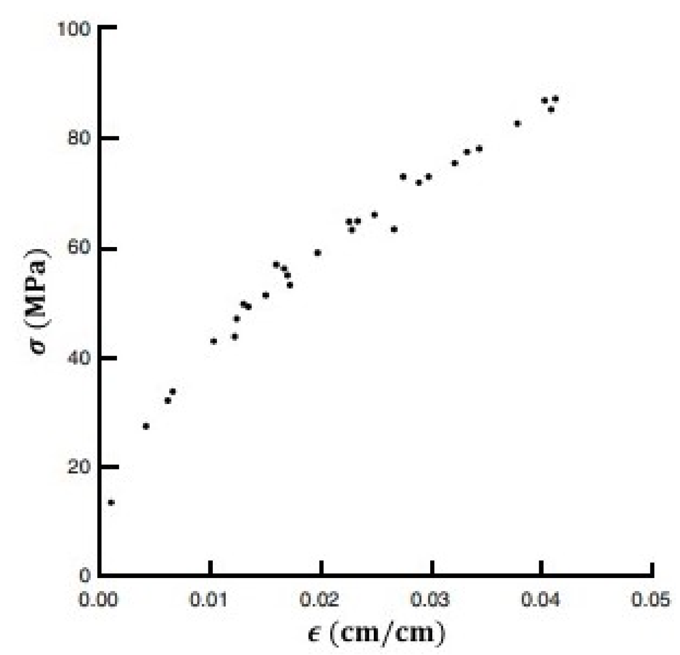

Figure 2: A typical data set for a truss bar [1].

Solution based on model-driven approach

We use the Hooke's Law model, where the Young's modulus for each bar is deduced by **data-fitting** some experimental data. From (1) we then obtain two linear equations for u_1 and u_2:

$$f_i = \sum_{e=1}^{3} V_e B_{ei} E_e \sum_{j=1}^{2} B_{ej} u_j \quad (\text{for } i = 1, 2).$$

Once the displacements are known, we use $(1)_1$ to compute the strains and then the stresses in the bars.

Solution based on data-driven approach

Here we **minimise the distance** between the experimental data and solution, subject to kinematics and equilibrium conditions. That is, we solve:

$$F_e(\varepsilon_e, \sigma_e) = \min_{(\varepsilon_e', \sigma_e')\epsilon D_e} \left\{ \frac{1}{2}C_e(\varepsilon_e - \varepsilon_e')^2 + \frac{1}{2C_e}(\sigma_e - \sigma_e')^2 \right\},$$

subject to $(1)_2$ using the method of Lagrangian multipliers. This gives 3 equations for the Lagrangian multiplier, σ_e and u_i. We then solve using iterations until the solution converges to the minimum. Note D_e is the experimental data for the e-th bar and C_e is a constant say, the typical stiffness.

Machine learning and next steps

ML is the study of computer algorithms focusing on "learning from experience" and improving accuracy over time [2]. We could run an algorithm on a training data set, train the algorithm to create a model and then improve the model through usage. Suppose for each input of forces we can find the displacement. But for any new input of forces, can we obtain solutions through ML?

References

[1] Kirchdoerfer, T., Ortiz M. Data-driven computational mechanics. *Computer methods in applied mechanics and engineering*. 2016. **304**, pp. 81-101.

[2] IBM Cloud Education. *What is Machine Learning?* [Online]. 2020. [Accessed 20 January 2021]. Available from: `https://www.ibm.com/`