Possible Topics of Master Projects

Victor I. Shrira

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1 'Black holes' for inertia-gravity waves in the uniformly stratified fluid at the critical latitude

The inertia-gravity waves are waves for which the restoring force is caused primarily by the Earth's rotation. Their typical periods and lengths at mid latitudes are respectively ~ 20h and $\sim 10^2$km. They play an important, although yet not fully understood role in dynamics of atmosphere and ocean. Crucially they enhance mixing. They can propagate poleward only up to a certain "critical" latitude (see Fig. 1).

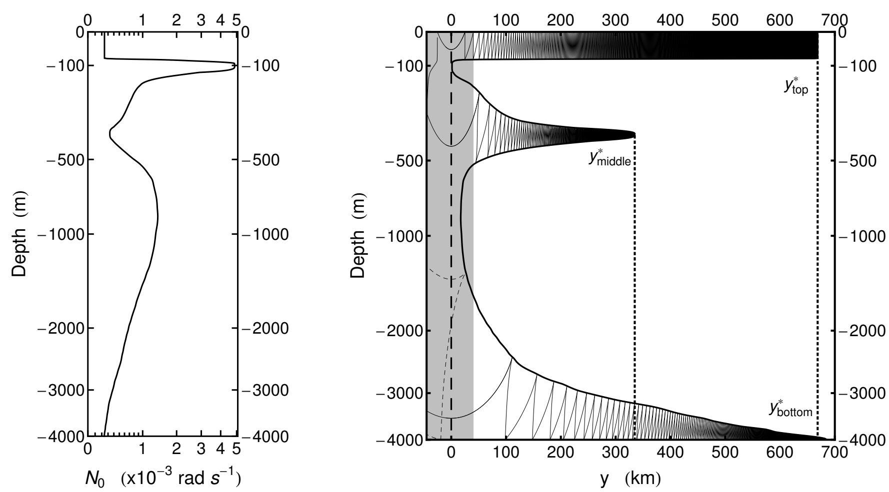

Figure 1: (a) A typical ocean stratification profile $N(z)$. (b) Thick solid line indicates boundary of the hypebolicity/ellipticity domains for an inertia-gravity wave of frequency σ_0. Thick dashed vertical line indicates the inertial latitude where $\sigma_0 = f$, (f is the Coriolis parameter) y is counted from the inertial latitude. Characteristics are shown in thin lines: thin solid – characteristics going through, thin dashed – reflected characteristics. Thick dotted lines indicate the positions of the tip latitudes their coordinates are denoted by y^*. Solution is sought to the right of the grey zone.

The figure shows behaviour of the characteristics of the governing equation. So far analytic solution for wave field in the vicinity of the critical latitude were obtained only for the lower and the middle waveguides [1]. The question what happens with these waves at the critical latitude (CL) remains open for the waves in the mixed layer of the ocean (the domain adjacent to the ocean surface), where density stratification is very weak and does not depend on depth. There are several possibilities. The most likely and the most interesting one is that the critical latitude acts as a '$b$lack hole' : all incident waves are absorbed and nothing penetrates beyond this latitude and nothing reflects back. At the moment this is a hypothesis. There is an asymptotic procedure which suggests that this is indeed the case, but this procedure breaks down in the vicinity of the critical latitude and what happens there nobody knows. To address the question one has to solve the known governing second order linear PDE of mixed (hyperbolic/ellptiic type) in the vicinity of the singularity. In Fig.1 the domains of hyperbolicity and ellipticity are separated by the bold solid line, the characteristics in the hyperbolicity domain are shown by thin solid lines. The challenge is to find analytical solution describing evolution of inertial wave near the critical amplitude. The results will advance understanding of an important aspect of atmosphere and ocean dynamics, they will also have implications for astrophysics.

Mathematically the project is in solving analytically a second order linear PDE and analysing the obtained solution. The fully accomplished project will constitute a foundation for a paper in a top rate journal.

References

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2 Generation of inertial waves in the subsurface layer of the ocean by a drop of wind

The waves of inertial period, i.e. waves with the frequencies $\sigma \approx f$, where f is the Coriolis parameter ($f = 2\Omega\varphi$, where φ is the local latitude, play important role in the dynamics of the ocean. They are of special interest in the context of mixing in the uppermost few meters of the ocean which plays crucial role in the ocean/athmosphere energy and momentum exchange. How these waves are generated is not clear. It is pretty obvious that they are generated by wind, but at present the specific mechanisms of their generation and their relative importance are unknown. The aim of the project is to examine one of the most plausible mechanisms.

It is known that wind generates a boundary layer current, called the *Ekman current*, which is adjacent to the ocean surface. Also adjacent to the surface of the ocean is its part called the *mixed layer*. It was recently suggested [1] that this mixed layer represents a waveguide for the waves of inertial range, which was later confirmed by observations [2]. There are observations suggesting that inertial waves are much more frequent after a drop of the wind. The this project aims at finding the underlying physical mechanism and developing a mathematical model of the phenomenon. The idea the examine is as follows The idea this project is supposed to examine is as follows. The Ekman current is a forced motion, that is, it is the motion caused by the wind and existing only as long as the wind blows. Thus, if there was a steady wind which generated a steady Ekman current, the current has to cease to exist when the wind drops. What then happens with the moving mass of water? The hypothesis is that at this moment inertial waves are generated. The challenge is to describe mathematically this process.

From the mathematical viewpoint the study will be concerned with a linear second order PDE. The most nontrivial element of the task is to "merge" the mathematical description of the Ekman currents and inertial waves which are commonly described by different models. The successful project will constitute a foundation for a paper in a top rate journal. In the long term the results might be incorporated into weather forecasting.

References

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3 Nonlinear evolution of edge waves

The edge waves are a class of low frequency water waves trapped by the bottom nearshore. They are important for predicting storm surges and sediment transport. They might be also an element of a meteotsunami. Currently most of theoretical models of edge waves are confined to linear models. This project aims to derive a weakly nonlinear evolution equation for long edge waves which will capture the essential physics and will be more tractable than the full Euler equations.

The idea underpinning the derivation is typical of the evolution equations derived in other contexts: we look at weakly nonlinear waves in the range of scales where their dispersion is also weak. We assume that to leading order nonlinearity and dispersion are in balance. The resulting equation we are aiming to derive will differ from its celebrated counterpart for free water waves - the Korteweg-de Vries equation. In the simplest case I expect it to be the Ostrovsky equation (the rotation modified Korteweg - de Vries equation) for the surface elevation η (see [2])

$$\partial_\xi(\partial_t \eta + \alpha\eta\partial_\xi\eta + \beta\partial_\xi^3\eta) = \delta_\xi\eta,$$

which has never been considered in the context of edge waves. Here α, β and δ are constant coefficients determined by the bottom profile, ξ is the spacial independent coordinate in the direction of wave propagation in the frame of reference moving with the speed $\sqrt{gH}$, where H is the water depth at the shelf, g is the acceleration due to gravity. In more general case I expect a more complicated pesudo-differential equation in the spirit of [1]. Apart from the asymptotic derivation of the evolution equation, some analysis of its solutions is also expected.

The successful project will constitute a foundation for a paper in a good journal.

References

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4 Nonlinear seiches

Seiches are the standing waves observed in closed or semi-closed basins (lakes, bays, seas and man made basins). In nature seiches might be excited by earthquakes and winds. To describe accurately the seiches with periods exceeding ∼1h it is necessary to take into account the effects of Earth's rotation. The existing models of seiches are linear. The aim of this project is to examine nonlinear model of seiches in the simplest possible setting.

Dynamics of long nonlinear waves in rotating fluid of constant depth H propagating in the x and $-x$ directions is governed by the closed equation on component of velocity v perpendicular to the direction of wave propagation,

$$\partial_t^2 v - c_0^2 \partial_x^2 v + f^2 v - \frac{H^3}{3}\partial_x^2 \partial_t^2 v = \partial_t \left[\frac{\partial_t v \partial_x v}{f + \partial_x v}\right] + \frac{f}{2}\partial_x \left[\frac{\partial_t v}{f + \partial_x v}\right]^2,$$

where free surface elevation η and x-component of velocity u are expressed in terms of x-component of velocity v as follows

$$\eta = \frac{H}{f}\partial_x v, \qquad u = -\frac{\partial_t v}{f + \partial_x v},$$

f is the Coriolis parameter, $c_0 = \sqrt{gH}$ and g is the gravity constant. Steady solutions of this equation in the limit where the term $\frac{H^3}{3}\partial_x^2 \partial_t^2 v$ is negligible are well studied (see e.g.[1]), however, standing waves have never been examined.

The successful project will constitute a foundation for a paper in a good journal.

References

1. R. H. J. Grimshaw, L. A. Ostrovsky, V. I. Shrira, and Yu. A. Stepanyants (1998) *Nonlinear surface and internal gravity waves in a rotating ocean* **Surv. in Geophysics 19**, #4, 289-338.