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Results for absolute and convective instabilities:

In Moresco & Healey (2000b) the absolute/convective character of viscous disturbances to a family of mixed-convection profiles were determined. The flow is that due to placing a vertical isothermal plate parallel to a uniform stream held at a different temperature to the plate. Near the leading edge the flow is close to Blasius flow, but far downstream buoyancy effects dominate and the flow approaches the Ostrach similarity solution. A coordinate transform technique (which gives the Blasius and Ostrach similarity equations in the appropriate limits) was used to obtain the basic flow at intermediate distances for both heated and cooled plates relative to the stream. It was shown that the flow only becomes absolutely unstable when the buoyancy force is opposed to the freestream direction causing the boundary layer to separate. The amount of reverse flow needed to generate absolute instability for this flow was calculated, and new instability modes were found at relatively large Reynolds numbers.

The flow produced by a rotating disk in an otherwise still fluid (the rotating disk boundary-layer) is known to be absolutely unstable above a critical Reynolds number, and also in the inviscid limit. It had been assumed that the upper branch of the neutral curve for absolute instability would asymptote towards the finite wavenumber cut-off for absolute instability in the inviscid problem as the Reynolds number increases, but in Healey (2004b) this is shown not to be the case. It is found that the dominant saddle in the finite wavenumber inviscid regime is not the same saddle-point as the one in the viscous problem which produces the onset of absolute instability. The Reynolds number at which the switch in dominance occurs was determined. Asymptotic theories were developed showing that the effect of nonparallel terms on the local absolute instability was destabilizing near the upper branch of the neutral curve. (Nonparallel stability calculations require a measurement quantity to be defined, but it was shown explicitly using the asymptotic formulae that the neutral curve for nonparallel absolute instability is independent of measurement quantity). However, the dominant saddle at the critical Reynolds number for absolute instability does appear to remain dominant along the lower branch of the neutral curve, where the absolute instability enters the long-wave regime.

A long-wave inviscid asymptotic theory for the absolute instability of the rotating-disk boundary-layer is developed in Healey (2006a). An explicit formula for the growth rate of the absolute instability as a function of basic flow parameters was obtained. The fundamental cross-flow structure responsible for the absolute instability in this limit was discovered: the flow is dominated by the radial wall-jet component, but a small reverse flow in the freestream due to the azimuthal component is also required. In addition, a curious behaviour of the dominant saddle-point, unnoticed in previous investigations, was revealed by the asymptotic theory (and subsequently verified by numerical stability calculations). The dominant saddle-point was found to approach the imaginary axis of the complex radial wavenumber plane. The integration contour only remains within the valleys of his saddle (i.e. between the pinching spatial branches) if it crosses this imaginary axis.

This appears to be problematic because branch-cuts are placed on the imaginary axis to enforce the exponential decay of disturbances in the wall-normal direction. An integration contour that crosses the imaginary axis (the branch-cut would have to be moved away from the imaginary axis to allow this) would utilise waves that grow exponentially in the wall-normal direction, thus failing to satisfy homogeneous boundary conditions in the wall-normal direction. This seems to be a physical argument for disallowing such a movement of the branch-cut, so that only waves decaying exponentially with wall-normal distance are used. However, from a mathematical view-point, the physical solution does not depend on the location of branch-cuts, so contours crossing the imaginary axis are allowable, and can not lead to a violation of boundary conditions.

This apparent paradox was resolved in Healey (2006b), where it is shown that a saddle-point approaching the imaginary axis in such a way that contours cross the imaginary axis do indeed produce disturbances that grow exponentially with distance from the wall, but (like leaky waves) only over a finite distance at any finite time. Beyond this distance the disturbance decays and always satisfies homogeneous boundary conditions. However, unlike leaky waves, the distance over which exponential wall-normal growth occurs increases with time, so that disturbances resemble a convective instability in the wall-normal direction. In a sense, this completes the 'signalling problem'. As the imaginary part of the frequency is reduced, a mode in the complex wavenumber plane that crosses from the upper half-plane to the lower half-plane is a downstream propagating unstable wave, and if the mode then crosses from the right half-plane to the left half-plane (as in the rotating-disk boundary-layer) it is a wall-normal propagating unstable wave. As with leaky waves, the source of energy for this exponential growth outside the boundary layer (where there is no Reynolds stress) can be found by considering simultaneously the propagation characteristics in both streamwise and wall-normal directions. A long-wave theory for the wall-normal propagation is presented in Healey (2005), which identifies the generic properties of the behaviour in the long-wave regime and shows explicitly how these properties depend on basic flow parameters.

If a flow has this type of wall-normal propagation, then it will be profoundly affected by placing a plate parallel to the flow, no matter how far away the plate is placed. This is because confining the flow causes the branch-cut at the imaginary axis to be replaced by an infinite discrete spectrum, and the wall-normal propagation is controlled by the dispersion relation properties at the imaginary axis. Healey (2007) shows that the effect of the outer plate is to replace the wall-normal propagation with a standing wave in the wall-normal direction that has stronger absolute instability than the original unconfined flow. Confinement usually has a stabilizing effect, but the opposite is true for this class of flows.

This behaviour of the saddle-point had been observed in other flow problems too, and these results provide the physical interpretation, and indicate how confinement can enhance absolute instability, or even create absolute instability. One classical problem where the saddle point behaves in this manner is the swirling jet. Healey (2008) shows how confinement can create an absolute instability of inviscid axisymmetric waves in the swirling jet, where there is no absolute instability of axisymmetric waves in the unconfined problem. This provides an intriguing possible explanation of axisymmetric vortex breakdown, since the appearance of an absolute instability is analogous to the super-critical sub-critical transition suggested by Benjamin.

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